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NRL Report 7991

An Analysis of the Breakup of Satellite 1974-103A (Cosmos 699)

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Space Systems Division

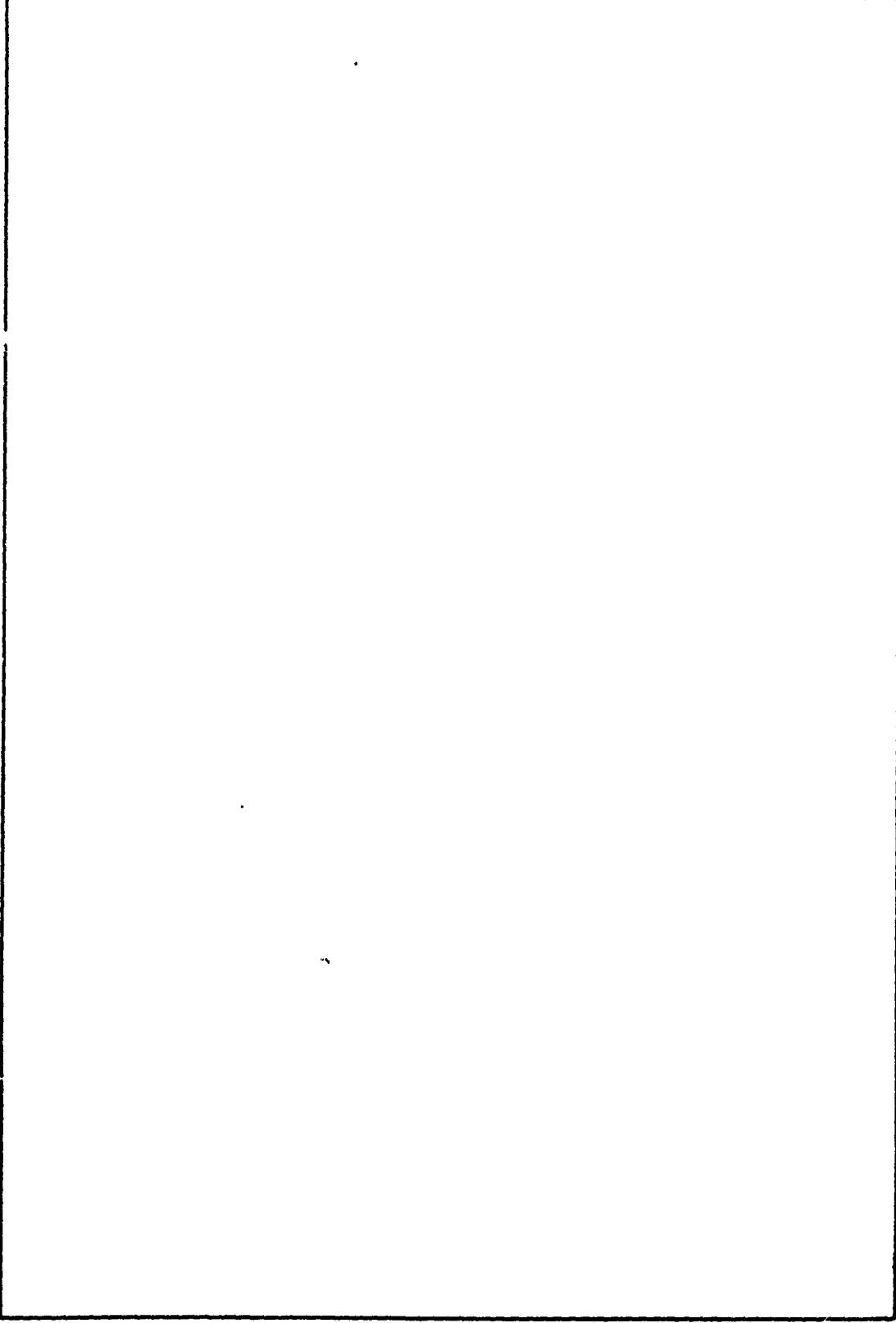
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CONTENTS

| | |
|----------------------------------|----|
| INTRODUCTION | 1 |
| THE OBSERVATIONAL DATA | 1 |
| THEORETICAL DEVELOPMENT | 7 |
| ANALYSIS OF FIRST MOMENTS | 10 |
| ANALYSIS OF SECOND MOMENTS | 12 |
| CONCLUSIONS | 13 |
| ACKNOWLEDGMENTS | 15 |
| REFERENCES | 17 |

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AN ANALYSIS OF THE BREAKUP OF SATELLITE 1974-103A (COSMOS 699)

INTRODUCTION

On 17 April 1975 the Department of Defense Space Surveillance System (NAVSPASUR, operated by the Navy, and SPADATS, operated by the Air Force) observed that satellite 1974-103A (Cosmos 699) had disintegrated into several fragments. According to the NASA Goddard Satellite Situation Report, SPADATS identified 50 objects associated with this breakup. Orbits for 38 fragments were determined by NAVSPASUR in the course of their analysis of the event.

The breakup of Cosmos 699 occurred as the NRL Systems Research Branch of the Space Systems Division was investigating the dynamics of ensembles of particles such as would result from a satellite breakup. The investigation had shown that the application of statistical mechanics, as done in the continuum theory of stellar dynamics, was a particularly promising approach. The interest surrounding the Cosmos 699 breakup has made it a focal point for the testing of the newly developed statistical theory of satellite breakups.

This report describes an analysis of the Cosmos 699 breakup base on the application of statistical mechanics and using NAVSPASUR elements for observational data. A guiding principle of the analysis is to contrast at every opportunity results obtained from the individual elements and those obtained from the statistical structure of the cloud as a whole.

THE OBSERVATIONAL DATA

The analysis is based on orbital elements of individual fragments which were determined at NAVSPASUR Headquarters, Dahlgren, Virginia. Thirty-eight element sets were supplied. Of these, three were rejected because of anomalous mean motion rates, and two were rejected because the time of epoch was considered to be either too near or too far from time of break-up. Thus 33 element sets were retained for analysis. The elements are listed in Table 1. Element set 1 is for the original COSMOS 699 payload. This element set provides the reference or parent orbit for the analysis.

The NAVSPASUR differential corrections (DC's) which estimated the elements were based on direction-cosine measurements of the NAVSPASUR "fence." The average number of observations per DC was ten over a time span of 3 days. Roughly 2/3 of the observations were single-station direction-cosine measurements, and roughly 1/3 involved two or more stations, which permitted a determination of range by triangulation. The DC for each

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Table 1 — Orbital Elements

| No. | To (days) | Mean Anomaly (deg) | Mean Motion (rev/day) | Decay Coef. (rad hr ⁻²) | Eccentric- ity | Argu- ment of Perigee (deg) | Longitude of Ascending Node (deg) | Inclina- tion (deg) |
|-----|--------------|--------------------------|-----------------------------|---|-------------------|--------------------------------------|--|---------------------------|
| 1 | 110.0 | 335.5510 | 15.4454 | 0.0051 | 0.00100 | 310.6776 | 56.3094 | 65.0404 |
| 2 | 110.0 | 359.0504 | 15.4551 | 0.0122 | 0.00048 | 279.2321 | 56.3158 | 65.0173 |
| 3 | 110.0 | 64.5858 | 15.3922 | 0.0155 | 0.00175 | 181.2430 | 56.3451 | 64.9356 |
| 4 | 110.0 | 335.3910 | 15.2546 | 0.0088 | 0.00782 | 166.9099 | 56.4975 | 64.9342 |
| 5 | 110.0 | 15.3752 | 15.3317 | 0.0030 | 0.00425 | 185.3580 | 56.3897 | 64.8417 |
| 6 | 110.0 | 264.2435 | 15.1835 | 0.0139 | 0.01078 | 184.6668 | 56.5601 | 64.9101 |
| 7 | 110.0 | 41.6927 | 15.3819 | 0.0183 | 0.00221 | 196.3839 | 56.3644 | 64.9645 |
| 8 | 110.0 | 57.0992 | 15.4052 | 0.0360 | 0.00131 | 197.9745 | 56.3354 | 64.9681 |
| 9 | 110.0 | 170.2737 | 15.4213 | 0.0057 | 0.00079 | 96.5992 | 56.2871 | 64.8971 |
| 10 | 110.0 | 182.5328 | 15.4322 | 0.0455 | 0.00047 | 92.5385 | 56.2981 | 64.9160 |
| 11 | 110.0 | 49.9472 | 15.4084 | 0.0104 | 0.00119 | 208.2318 | 56.3392 | 64.9862 |
| 12 | 110.0 | 55.4496 | 15.4031 | 0.1043 | 0.00155 | 196.1986 | 56.3412 | 64.9674 |
| 13 | 110.0 | 348.7623 | 15.4378 | 0.0122 | 0.00088 | 291.4285 | 56.3201 | 65.0201 |
| 14 | 110.0 | 98.6550 | 15.4135 | 0.0594 | 0.00097 | 161.9419 | 56.3186 | 64.9234 |
| 15 | 110.0 | 102.7159 | 15.3919 | 0.0923 | 0.00248 | 140.5360 | 56.3509 | 64.9434 |
| 16 | 110.0 | 42.8998 | 15.3886 | 0.0799 | 0.00195 | 198.3201 | 56.3573 | 64.9654 |
| 18 | 110.0 | 302.4745 | 15.1966 | 0.0226 | 0.01107 | 155.5604 | 56.4660 | 64.5325 |
| 19 | 110.0 | 71.0607 | 15.4077 | 0.0107 | 0.00102 | 186.5389 | 56.3369 | 64.9764 |
| 20 | 110.0 | 53.9622 | 15.4073 | 0.0402 | 0.00126 | 202.5364 | 56.3382 | 64.9787 |
| 21 | 110.0 | 30.7012 | 15.4212 | 0.0447 | 0.00097 | 236.1496 | 56.3247 | 64.9862 |
| 22 | 110.0 | 48.6885 | 15.3858 | 0.0256 | 0.00207 | 191.9969 | 63.3047 | 64.9530 |
| 23 | 110.0 | 224.7948 | 15.1156 | 0.0039 | 0.01379 | 172.9877 | 56.6363 | 64.8813 |
| 24 | 110.0 | 57.5153 | 15.3871 | 0.1657 | 0.00274 | 180.0411 | 56.3196 | 64.9064 |
| 25 | 111.0 | 16.1099 | 14.8794 | 0.0036 | 0.02512 | 159.4454 | 52.7553 | 64.6755 |
| 26 | 111.0 | 254.3241 | 15.4095 | 0.0584 | 0.00125 | 148.3192 | 52.0097 | 64.9407 |
| 27 | 111.0 | 203.8913 | 15.4003 | 0.0226 | 0.00132 | 191.0535 | 51.9310 | 64.8473 |
| 28 | 111.0 | 293.2920 | 15.4276 | 0.0312 | 0.00069 | 130.9905 | 51.9967 | 64.9656 |
| 29 | 112.0 | 345.7332 | 15.4077 | 0.0820 | 0.00120 | 197.0296 | 47.6841 | 64.9417 |
| 30 | 111.0 | 5.8121 | 15.2187 | 0.0107 | 0.00939 | 187.8631 | 52.2603 | 64.7772 |
| 31 | 112.0 | 109.9456 | 15.4336 | 0.0951 | 0.00069 | 110.0842 | 47.6631 | 64.9636 |
| 32 | 112.0 | 319.2130 | 15.4304 | 0.0267 | 0.00107 | 262.6812 | 47.6601 | 64.9739 |
| 33 | 110.0 | 152.3584 | 15.0732 | 0.4037 | 0.01797 | 203.9177 | 56.5967 | 64.4484 |
| 34 | 110.0 | 44.1956 | 15.4261 | 0.0244 | 0.00068 | 226.9592 | 56.3232 | 65.0006 |

element set used estimated the classical elements and the rate of change of mean motion which parameterizes the atmospheric drag perturbation. The rms value of the residuals for the DC's was typically 500 to 1000 meters.

For the present analysis the element sets were propagated by calculating two-body and secular perturbative effects. This introduces an inconsistency, because the complete Brouwer theory was used in the DC process at NAVSPASUR. However the differences (certainly less than 1 km) are insignificant for the purposes of this analysis.

The elements were found to fall into two classes. The first class consisted of 28 fragments which dispersed simultaneously from the parent body at 107°.9083 (3.0°N, 82.9°W). The second class consisted of five fragments which apparently fragmented at different times. The second class will be discussed further in the concluding section.

The evolution of the positions of the fragments is shown in Figs. 1 through 8 during the first 1/2 revolution after breakup. The differences are resolved along the unit vectors

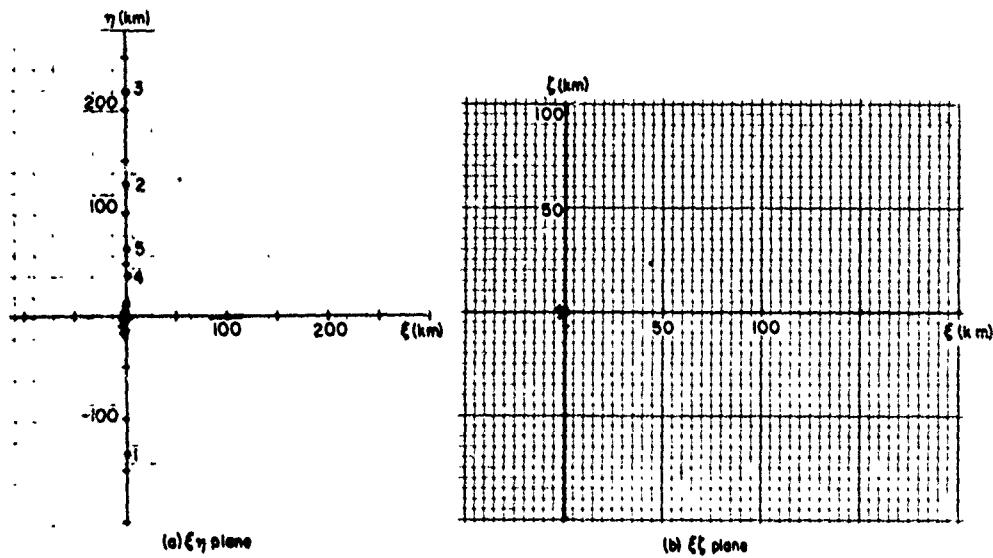


Fig. 1 — Position of fragments relative to the payload when $v - v_0 = 0.03$ radian (a) $\xi\eta$ plane (b) $\xi\epsilon$ plane. Fragments 1 through 5 correspond to element sets 9, 29, 31, 32, and 33 respectively (Table 1).

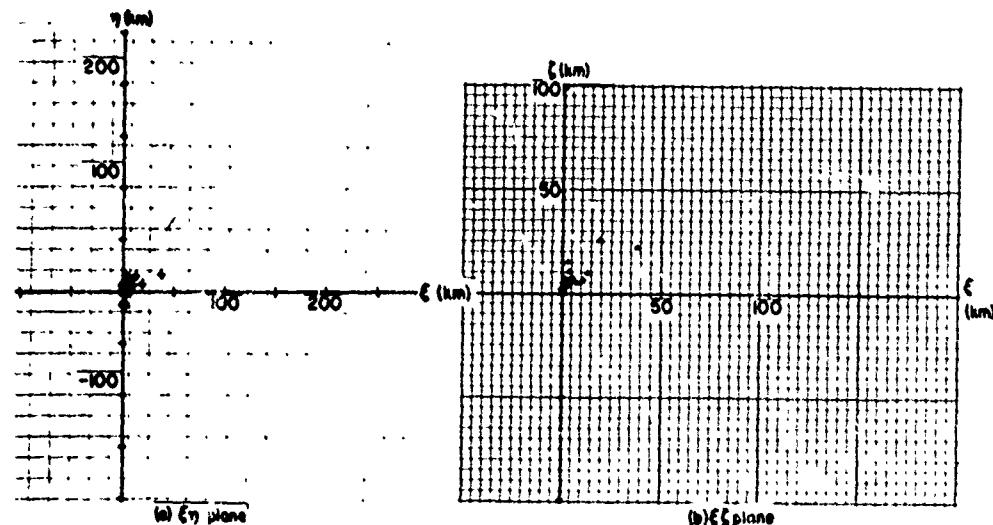


Fig. 2 — Position of fragments relative to the payload when $v - v_0 = 0.48$ radian
 (a) $\xi\eta$ plane (b) $\xi\epsilon$ plane

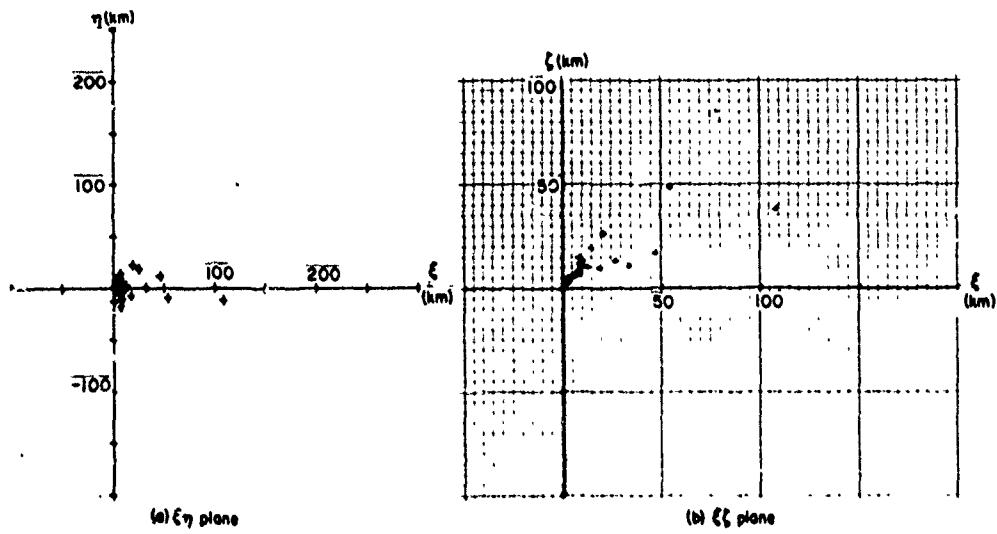


Fig. 3 — Position of fragments relative to the payload when $v - v_0 = 0.97$ radian
 (a) $\xi\eta$ plane (b) $\xi\xi$ plane

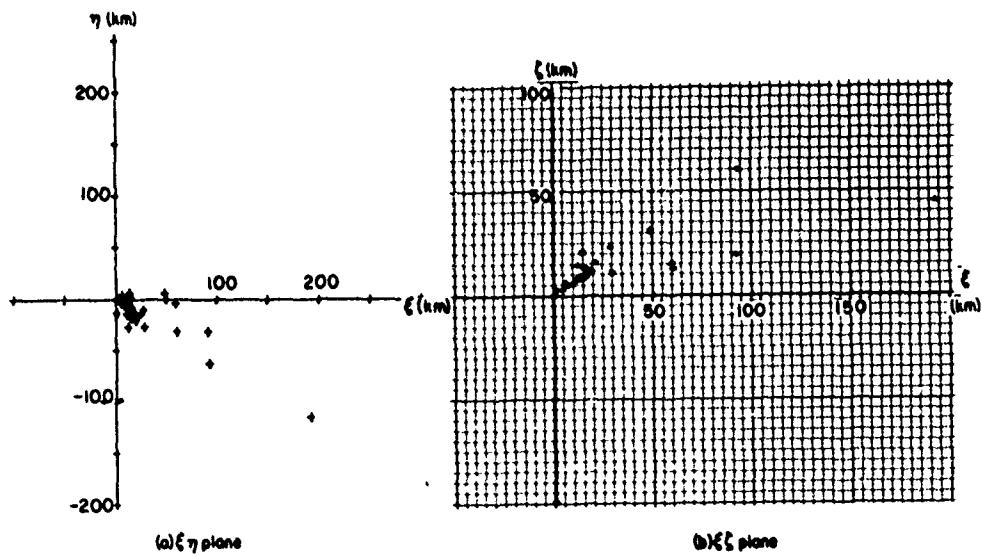


Fig. 4 — Position of fragments relative to the payload when $v - v_0 = 1.46$ radians
 (a) $\xi\eta$ plane (b) $\xi\xi$ plane

NRL REPORT 7991

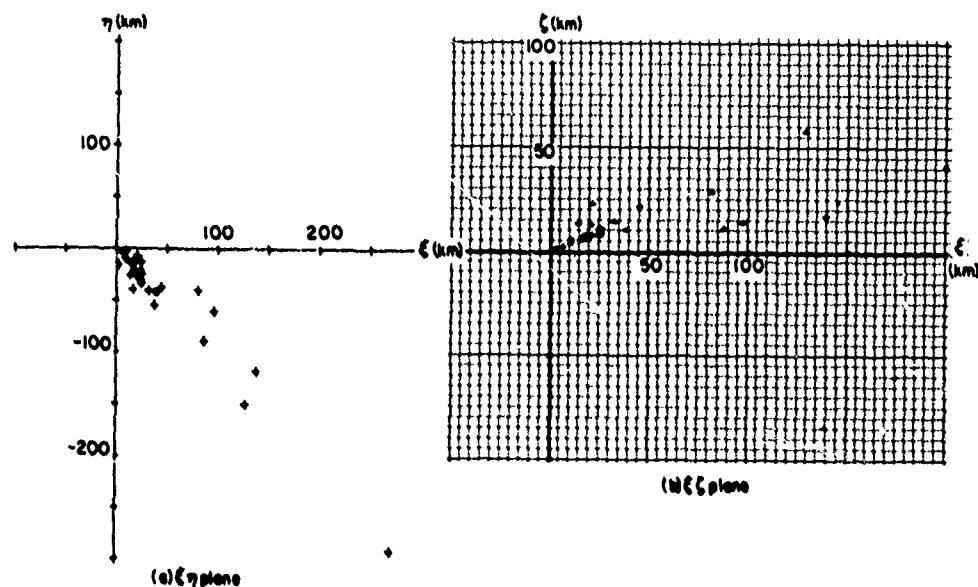


Fig. 5 — Position of fragments relative to the payload when $v - v_0 = 1.94$ radians
 (a) $\xi\eta$ plane (b) $\xi\zeta$ plane

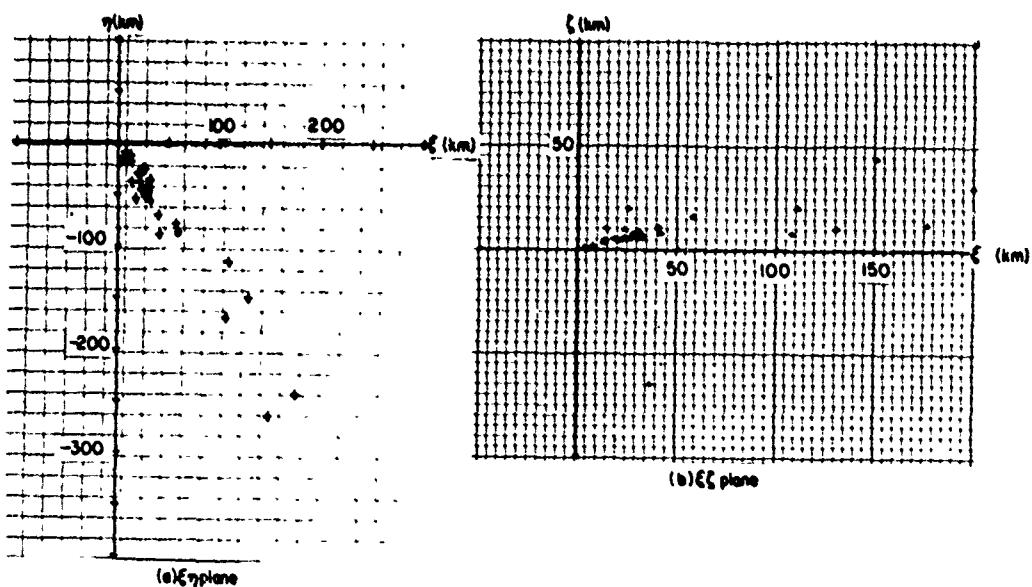


Fig. 6 — Position of fragments relative to the payload when $v - v_0 = 2.43$ radians
 (a) $\xi\eta$ plane (b) $\xi\zeta$ plane

W.B. HEARD

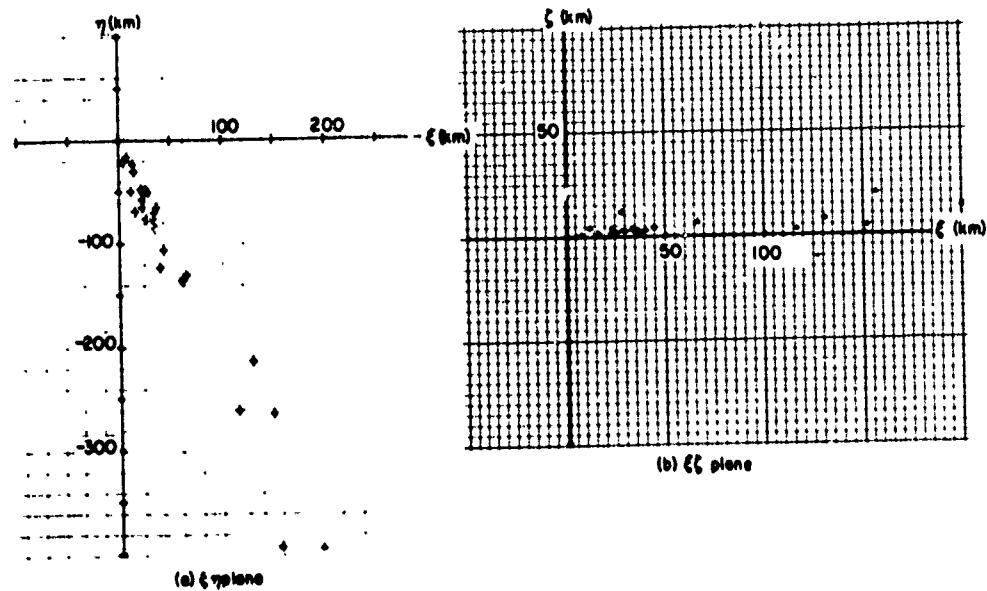


Fig. 7 — Position of fragments relative to the payload when $v - v_0 = 2.91$ radians
 (a) $\xi\eta$ plane (b) $\xi\xi$ plane

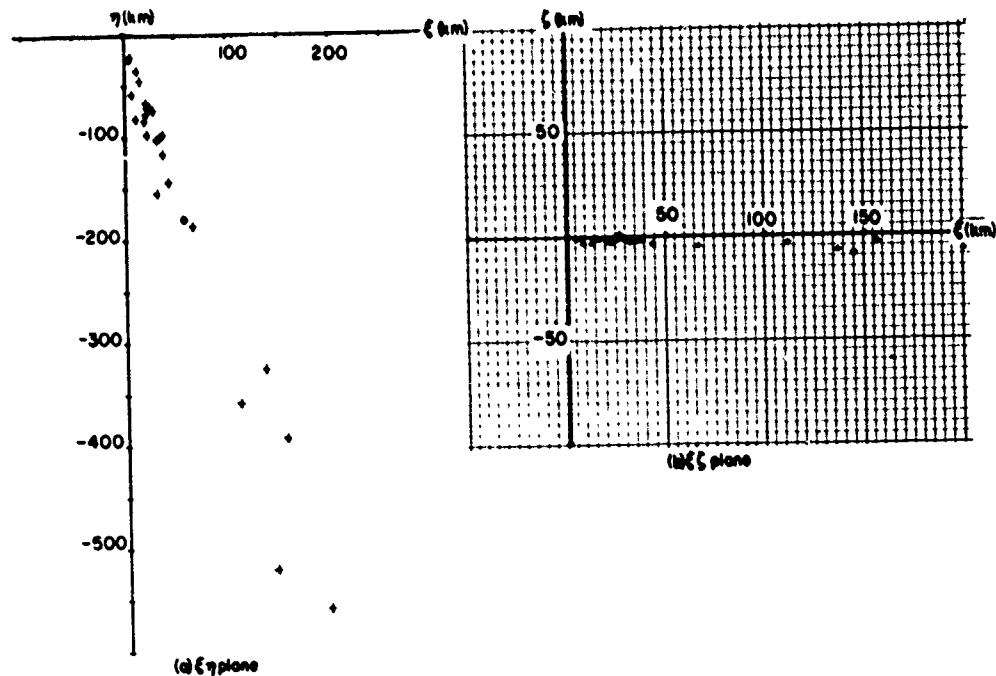


Fig. 8 — Position of fragments relative to the payload when $v - v_0 = 3.40$ radians
 (a) $\xi\eta$ plane (b) $\xi\xi$ plane

NRL REPORT 7991

$$\begin{aligned} \mathbf{e}_1 &= \mathbf{r} / |\mathbf{r}| && (\xi \text{ component}), \\ \mathbf{e}_3 &= \mathbf{r} \times \mathbf{v} / |\mathbf{r} \times \mathbf{v}| && (\zeta \text{ component}), \\ \mathbf{e}_2 &= \mathbf{e}_3 \times \mathbf{e}_1 && (\eta \text{ component}), \end{aligned}$$

where \mathbf{r} and \mathbf{v} are the position and velocity vectors of the parent. Figures 1a through 8a show the projection of the relative positions onto the $\xi \eta$ plane, and Figs. 1b through 8b show the projections onto the $\xi \zeta$ plane. The five fragments of the second class are indicated by the numbered filled circles in Fig. 1a.

THEORETICAL DEVELOPMENT

The theoretical basis for this analysis is the application of statistical mechanics to study ensembles of noninteracting particles [1]. There is however an area of the theory which must be expanded before we can proceed with the analysis. The inverse problem of determining the age and initial velocity distribution has been treated in detail only for a two-dimensional ellipsoidal breakup from a circular parent orbit. This must be extended to three dimensions in order to proceed. Thus this section is devoted to the inverse problem for a slowly dispersing three-dimensional ellipsoidal breakup. There is no reason for restriction to circular parent orbits in the theoretical development, but the eccentricity of Cosmos 699 was small enough that this simplification can be used in the application.

Statistical mechanics theory treats the ensemble of fragments as a continuum of particles described by a phase-space distribution function $f(\mathbf{q}, \mathbf{p}, t)$. If the equations of motion of the individual fragments are determined by the Hamiltonian $\mathcal{H}(\mathbf{q}, \mathbf{p}, t)$, then f satisfies Liouville's equation

$$\frac{\partial f}{\partial t} + \{f, \mathcal{H}\} = 0, \quad (1)$$

where (\cdot, \cdot) denotes the Poisson bracket. The initial condition satisfied for a body disintegrating at coordinate q_* at $t = 0$ is

$$f(\mathbf{q}, \mathbf{p}, 0) = \delta(\mathbf{q} - \mathbf{q}_*) G(\mathbf{p}), \quad (2)$$

where $\delta(\cdot)$ is the Dirac delta function and $G(\mathbf{p})$ describes the initial distribution of momenta.

The equations of motion may be linearized about the reference trajectory if the particles disperse slowly or, equivalently, if the concern is with the initial phase of the breakup. Then the solution of the equations of motion becomes

$$\begin{pmatrix} \mathbf{q} \\ \mathbf{p} \end{pmatrix} = \begin{pmatrix} U & V \\ W & Y \end{pmatrix} \begin{pmatrix} \mathbf{q}_0 \\ \mathbf{p}_0 \end{pmatrix}. \quad (3)$$

As shown in Ref. 1 the solution for f becomes

$$f = \delta(U_q + V_p - q_*) G(W_q + Y_p), \quad (4)$$

W.B. HEARD

and the spatial density

$$\rho(\mathbf{q}, t) = \int f(\mathbf{q}, \mathbf{p}, t) d\mathbf{p} \quad (5)$$

becomes

$$\rho = \det(V_-^{-1}) G[(W_- - Y_- V_-^{-1} U_-) \mathbf{q} + Y_- V_-^{-1} \mathbf{q}_*], \quad (6)$$

where the notation $U(-t) = U_-(t)$, etc., is used. We may set $\mathbf{q}_* = 0$, because the breakup originates on the parent orbit.

The momentum distribution for an ellipsoidal breakup may be written

$$G(\mathbf{p}) = \text{const } e^{-(\mathbf{p} - \mathbf{p}_0) \cdot A(\mathbf{p} - \mathbf{p}_0)}. \quad (7)$$

The problem is to find the mean momentum \mathbf{p}_0 and the symmetric dispersion matrix A given the position of the fragments relative to the parent body as a function of time. To do this, the spatial density function is written as

$$\rho = K(t) e^F \quad (8a)$$

with

$$K = (\det V_-)^{-1}, \quad (8b)$$

$$F = \mathbf{u} \cdot Q \mathbf{u}, \quad (8c)$$

$$\mathbf{u} = \mathbf{q} - M^{-1} \mathbf{p}_0, \quad (8d)$$

$$Q = M^T A M, \quad (8e)$$

and

$$M = W_- - Y_- V_-^{-1} U_-. \quad (8f)$$

The initial mean momentum is obtained immediately as

$$\mathbf{p}_0 = M \bar{\mathbf{q}},$$

where $\bar{\mathbf{q}}$ is the mean of the relative positions at time t . To obtain A , the matrix J is defined by

$$\text{ent}_{ij} J = \int u_i u_j \rho du. \quad (9)$$

This matrix is the point of contact with the observations and is obtained by calculating the second moments about the mean of the relative coordinates of the fragments. The theoretical value of the integral (9), assuming the ellipsoidal distribution (7), then relates A to the observables. If (8a) is substituted into (9), then [2] the result is

$$J = \frac{1}{2} \pi^{3/2} K (\det Q)^{1/2} Q^{-1}. \quad (10)$$

The determinant of (10) yields the relation

$$(\det Q)^{-1/2} = \left(\frac{2}{\pi^{3/2} K} \right)^{3/5} (\det J)^{1/5}. \quad (11)$$

Therefore

$$Q = \left(\frac{\pi^{3/2} K}{2} \right)^{2/5} (\det J)^{1/5} J^{-1}. \quad (12)$$

Finally, from (8e),

$$A = (M^{-1})^T Q M^{-1}, \quad (13)$$

and this is the expression relating A to the observable J via equation (12).

The expressions (3) through (13) are valid for any linear dynamical system. The analysis here of Cosmos 699 however will be restricted to linearization about a circular Keplerian orbit. In this case explicit expressions for M^{-1} and K are

$$M^{-1} = \frac{1}{n} \begin{pmatrix} s & 2(1-c) & 0 \\ 2(c-1) & 4s - 3nt & 0 \\ 0 & 0 & s \end{pmatrix}$$

and

$$K = \frac{n}{|s[-3nts + 8(1-c)]|}$$

where $s = \sin nt$, $c = \cos nt$, and n is the mean motion of the parent orbit. Equation (11) also provides an algorithm for estimating the age of the breakup. From equation (8e)

$$(\det Q)^{-1} \propto \det M^{-1}.$$

Therefore from equation (11)

$$(\det J)^{1/5} \propto \left(\frac{\pi^{3/2} K}{2} \right)^{3/5} (\det M^{-1}). \quad (16)$$

The left-hand side of (16) is obtained directly from the observations as a function of clock time. The right-hand side is a known function of time since breakup. The time of breakup is determined by translating the left-hand side in time to match the right-hand side. The translation determines time since breakup and therefore determines the breakup time itself. Since the reference orbit is known, knowledge of breakup time immediately provides the breakup position.

W.B. HEARD

ANALYSIS OF FIRST MOMENTS

In this section, which begins the analysis of the Cosmos 699 data, the first moments, or means, of the relative positions of the fragments are analyzed. According to the theory in its linear approximation, the center of mass of the fragment ensemble will execute an epicycle in the reference plane. The out-of-plane motion is harmonic and is decoupled from the in-plane components. The observed epicyclic/harmonic motion will be used to determine both the mean of the initial velocity components and breakup age. The virtue of analyzing the fragment ensemble as a whole as opposed to straightforward statistics derived from the NAVSPASUR elements is that the former minimizes the effect of error that is inevitably present in the orbit-determination process. The velocity differences at breakup are comparable to the error expected in individual elements. However the extent of the ensemble cloud approximately 1/4 revolution after breakup far exceeds the error expected in the elements. Thus one can obtain reliable direct estimates of the structure of the fragment ensemble as opposed to unreliable direct estimates of the velocity differences at breakup time. The statistical theory enables one to translate the former into information about conditions at breakup time.

The mean of the ξ component versus time is shown in Fig. 9. The data fit the sine curve

$$\bar{\xi} = 14.4 \sin(v - v_0 - 0.03).$$

The theoretical expression for this mean is

$$\bar{\xi} = (\bar{\xi}_0/n) \sin[n(t - t_0)].$$

The clock times associated with the difference in true anomaly $v - v_0$ and the mean motion of the parent ($n = 0.001123$ rad/s) yields

$$\bar{\xi}_0 = 16.2 \text{ m/s}$$

and

$$t_0 = 107^d.9083.$$

The mean of ξ versus the mean of η is shown in Fig. 10. The theoretical expression for this curve is given parametrically by

$$\bar{\xi} = (\bar{\xi}_0/n) \sin[n(t - t_0)] + (2\bar{\eta}_0/n) \{1 - \cos[n(t - t_0)]\}$$

and

$$\bar{\eta} = (-2\bar{\xi}_0/n) \{1 - \cos[n(t - t_0)]\} + (\bar{\eta}_0/n) \{4 \sin[n(t - t_0)] - 3n(t - t_0)\}.$$

The data are well fit by the theoretical curve with

$$\bar{\xi}_0 = 4 \text{ m/s}$$

NRL REPORT 7991

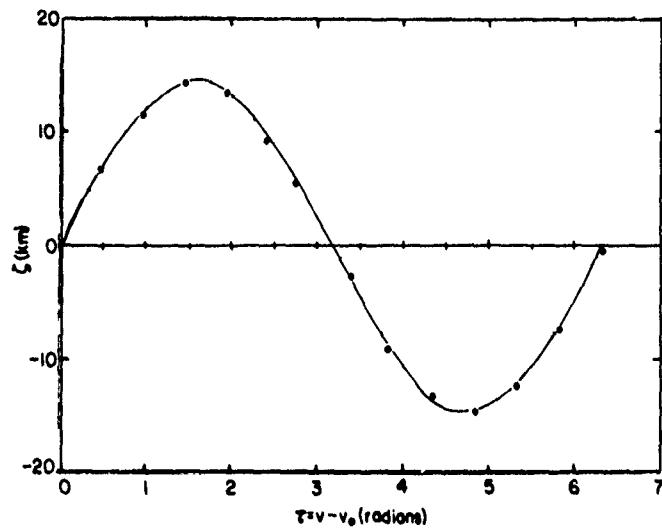


Fig. 9 — Average ξ coordinate versus true anomaly difference τ

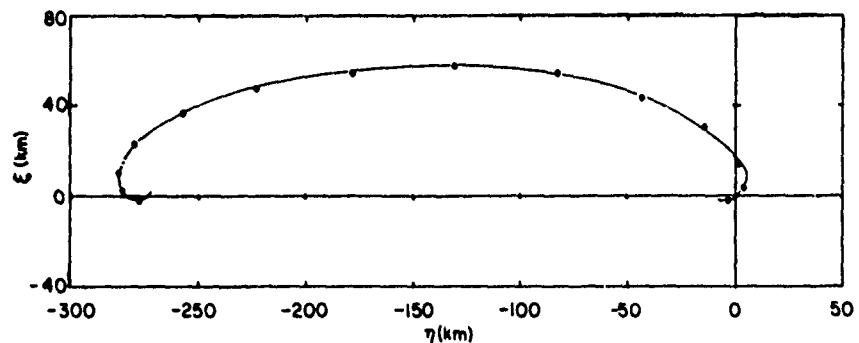


Fig. 10 — Average ξ coordinate versus the average η coordinate

and

$$\bar{\eta}_0 = 16.6 \text{ m/s.}$$

The amplitude of the sine curve and the epicycle are quite sensitive to ξ_0 and $\bar{\eta}_0$ respectively. Consequently the uncertainties in these values are less than 0.2 m/s. The value of ξ_0 is more difficult to estimate. The timing along the epicycle is the parameter most sensitive to ξ_0 . The value given is probably accurate to 1 m/s.

In Fig. 11 are shown the resolutions of the plane components of velocity residuals. Also shown are the estimated values of $\bar{\xi}_0$ and $\bar{\eta}_0$. They are seen to be consistent. However a straightforward calculation of $\bar{\xi}_0$ and $\bar{\eta}_0$ from these data produces an epicycle which fits the data poorly. Hence this is the first example of the advantage of the statistical theory with regard to observational error.

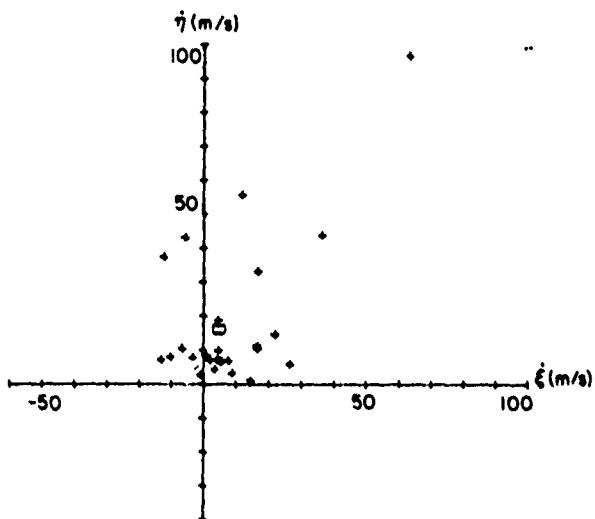


Fig. 11 -- Projection of the velocity difference at breakup time onto the $\xi\eta$ plane

ANALYSIS OF SECOND MOMENTS

The second moments contain a wealth of information. The breakup time may be inferred from the run of either the dimensions or the orientation of fragment ensemble. The velocity dispersions may be obtained by assuming, as here, an ellipsoidal distribution. These dispersions give some indication, based solely on dynamical considerations, of the mechanism responsible for the breakup. To see why this is plausible, one can consider the two extreme possibilities of intentional, on-board destruction and destruction from the impact of a projectile. The former could likely yield a nearly isotropic velocity distribution, and the latter should yield a highly directional velocity distribution indicative of the angle of approach of the projectile.

The velocity dispersions will be considered first. Figure 12 shows the reciprocal square roots λ_i of the eigenvalues of matrix A. This matrix was calculated from equations (12) and (13), the second moments of the matrix J having been calculated from the relative positions according to the NAVSPASUR elements. These eigenvalues are constants in the theory. The variability evidenced in Fig. 12 may be attributed to observational error and the departure of the actual spatial density from the assumed ellipsoidal form. The largest of the λ_i varies the least and may be estimated to be 4.25 with an uncertainty of ± 0.1 .

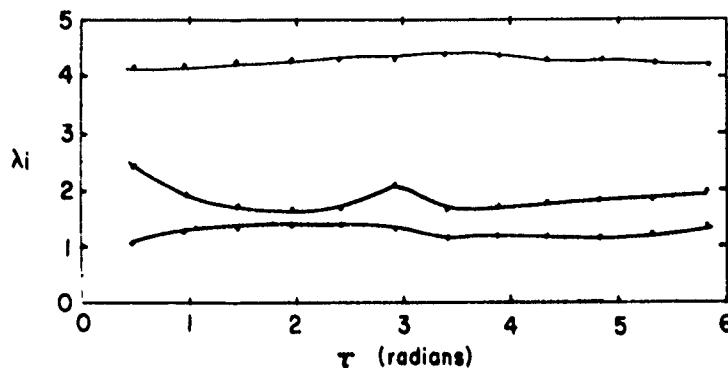


Fig. 12 — Reciprocal square roots of the eigenvalue of the calculated velocity dispersion matrix versus the true anomaly difference

(relative uncertainty 2.3%). The other two may be estimated to be 1.7 ± 0.3 and 1.20 ± 0.2 (relative uncertainty approximately 17% in both). This establishes the shape of the velocity ellipsoidal to be roughly a prolate spheroid with aspect ratio 2.8:1. The orientation of the spheroid is determined by the eigenvector associated with the largest λ_i . Figure 13 shows the orientation angles of that eigenvector in the $e_1 e_2 e_3$ frame. There is considerable scatter about the mean values $\theta = 31.1^\circ$ and $\phi = 76.5^\circ$. The scatter places the direction of this eigenvector inside a cone whose axis is in the mean direction and with an apex angle of about 10° . The scatter in the other two eigenvectors is too large to permit a meaningful estimate of their directions. This may be attributed to observational error and to the computational ambiguity of finding the principal axes of an ellipsoid which is nearly a spheroid. The dot product of the unit vector associated with the mean direction of the principal axis (indicated by a cross in Fig. 13), namely $e = (0.1927, 0.8233, 0.5168)$, and the unit vector associated with the mean velocity increment, namely $e = (0.1701, 0.7059, 0.6876)$, shows the angle between the two to be 12.4° . Thus the velocity ellipsoid is oriented roughly in the direction of the mean velocity increment. This shows that the Cosmos 699 breakup was highly directional. The mean velocity increment and the velocity ellipsoid are illustrated in Fig. 14.

The time of breakup follows from the second moments from both equation (16) and the discussion of it. A comparison of observed and calculated values for equation (16) are shown in Fig. 15. The amplitude is unimportant as far as the breakup time is concerned. This figure shows that the breakup time derived from the first moments is consistent with that obtained from the run of the dimension of the ensemble. No simple shift of origin would yield a better fit than shown.

CONCLUSIONS

First, some conclusions can be made with regard to the five outlying particles in Fig. 1a. The particle labeled 5 is merely a high-velocity particle ($\Delta v \approx 170$ m/s) which does

W.B. HEARD

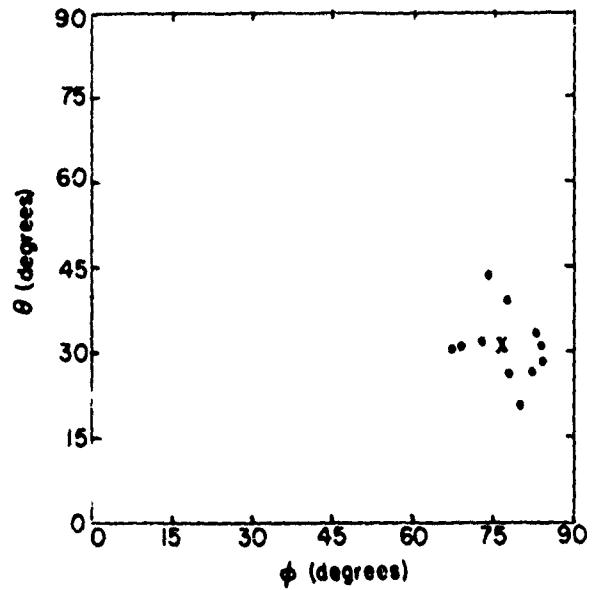


Fig. 13 — Orientation angles of the eigenvector associated with the largest λ_i

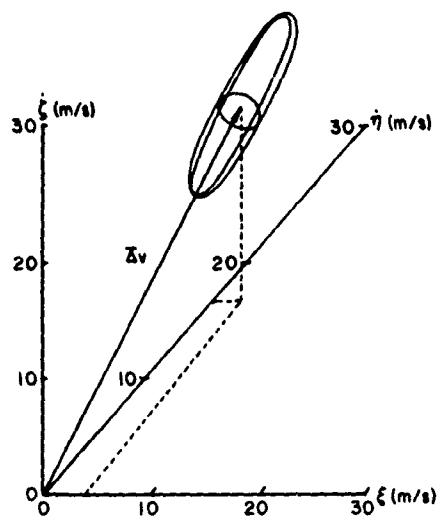


Fig. 14 — Illustration of the mean velocity increment and the velocity ellipsoid

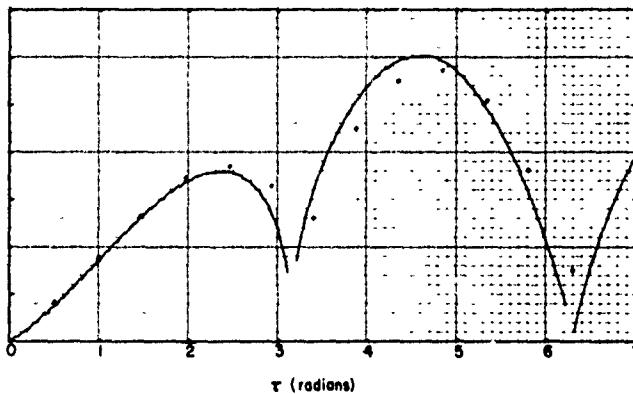


Fig. 15 — Function which determines the time of breakup from the matrix of second moments

originate with the main ensemble but evolves much more quickly. The remaining four particles however appear to have fragmented at different times. The relative trajectories of these particles (projected onto the $\xi\eta$ plane) are shown Fig. 16. The positions of the particles at breakup time are shown by large filled circles. When particle 1 is followed backward in time, it passes near the parent exactly two revolutions before breakup. If particles 2, 3, and 4 are followed forward, it is found that 2 and 4 pass near the parent exactly one revolution after breakup and particle 3 does so exactly three revolutions after breakup. It appears that these particles fragmented one and three revolutions respectively after the main breakup and did not even exist at breakup time. Their positions shown in Fig. 1a are purely fictitious. Another interesting observation is that all four epicycles leave roughly the same dimensions (as do their excursions out of reference plane) and that these in turn are typical of particles in the main ensemble. There is little doubt that they share a common fragmentation mechanism with the main ensemble. The scenario of a breakup is summarized on a ground trace in Fig. 17.

Second, some comments can be made with regard to the effectiveness of the theory. In spite of the facts that the eccentricity of the reference orbit is not zero (albeit small), that drag was included in the orbit calculations but not the statistical theory, and that a rather small sample size was available, the theory works remarkably well. An examination of Figs. 1a through 8a quickly convinces one that the actual spatial density is not ellipsoidal either, because of the apparent asymmetry along the major axis. Thus higher order moments would be necessary to describe the original velocity distribution in more detail. However, the computational difficulties encountered with the second moments indicates that one must anticipate trouble if such an attempt is made.

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W.B. HEARD

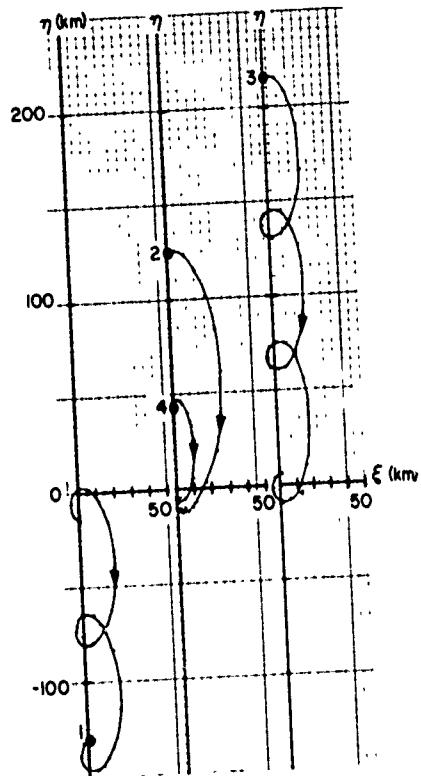


Fig. 16 — Positions of the exceptional fragments (Fig. 1a) relative to the payload as a function of time (projection onto the $\xi\eta$ plane)

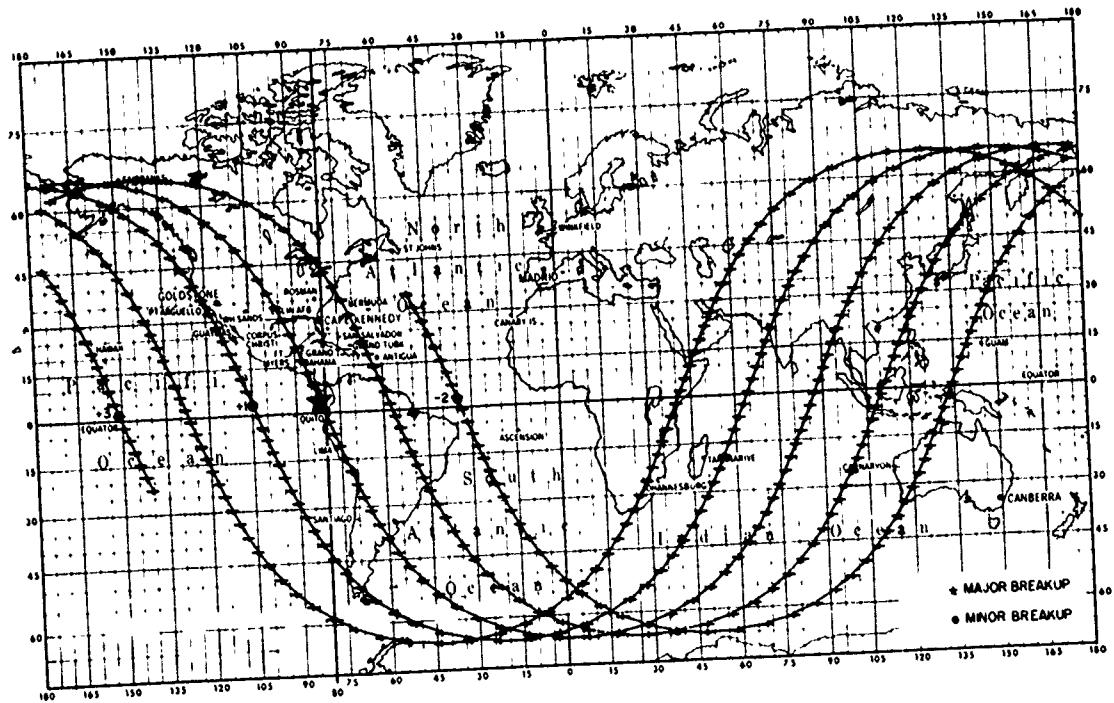


Fig. 17 — Ground trace for Cosmos 699

NRL REPORT 7991

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